Impulse Response of the UWB channel with perfectly and non perfectly conducting convex obstacles

Piotr Górniak[#1], Wojciech Bandurski[*2],

[#]Chair of Multimedia Telecommunication and Microelectronics, Poznań University of Technology, Poland

1pgorniak@et.put.poznan.pl

[2]wojciech.bandurski@put.poznan.pl

Abstract— **In the paper there is presented the 2D time-domain model of two bare convex obstacles model, which includes an application of an amplitude and the first order transition zone diffraction process, called slope diffraction. The considered model is represented by its impulse response. Thanks to the usage of some approximations the formulation for this impulse response can be presented in a closed form. The applied method of deriving the 2 bare convex obstacles model can be extended for the case of more cascaded convex obstacles in the channel. The method uses Uniform Theory of Diffraction (UTD) formulated in the frequency domain (FD). The accuracy of the derived model is examined in the process of calculation of an Ultra Wide Band (UWB) pulse distortion.**

I. INTRODUCTION

In the paper we deal with time domain (TD) modeling of a diffraction caused by convex objects. We do it for the purpose of deterministic UWB channel modeling for the channel with convex objects. Among the examples of such objects are round buildings, round pillars in buildings or round corridors in buildings. We consider the diffraction on convex objects for the soft polarisation case, when more than one such object are placed very close to each other in the way that one convex object is in the transition zone of another scattering object. This is the situation when slope diffraction is the key factor and can not be omitted. When the UWB signal propagation is taken into account the time domain modeling is the right choice.

The model of transition zone diffraction on convex objects was discussed in the frequency domain in [1]. In the paper we present the method of deriving the impulse response of two cascaded perfectly conducting convex obstacles shadowing transmitter and receiver. This impulse response is obtained in a close form. The proposed method can be extended for the cases of more tan 2 cascaded convex obstacles.

We present the impulse response of two conducting cascaded convex obstacles, which is based on the Uniform Theory of Diffraction (UTD) formulated in the frequency domain. The resulting impulse response is a sum of two terms, amplitude term and slope term, which is important in the transition zones. The amplitude term and slope term of the resulting impulse response are associated with the amplitude term and slope term of the TD convex obstacle diffraction coefficient respectively. We show two forms of the time domain convex obstacle diffraction coefficient, depending on the frequency band which occupies analyzed UWB pulse

The impulse response is used to examine the UWB pulse distortion caused by the channel. The time domain formulations is verified by comparing the results of the convolutions of the impulse response of the channel and an exemplary UWB pulse with the results of frequency domain calculations with the usage of inverse fast Fourier transform (IFFT).

The paper is organised as follows. In Section 2 we describe the model of a convex obstacles. Section 3 presents the TD version of the convex object diffraction coefficient. In Section 4 we show how to obtain the overall time domain field expression after two conducting cascaded convex obstacles and the method of solving the operations of convolution in the overall time domain field expression. It is done through applying some approximations and the usage of optimising algorithm (genetic algorithm). Section 5 gives the numerical results and the verification of the theoretical model. Finally, the summary and conclusions are presented in Section 6.

II. DEFINITION OF THE CHANNEL MODEL AND ITS TRANSFER FUNCTION

A convex 2D obstacle is modeled as a part of a circle, as shown in Fig. 1. This model was proposed by H. Bertoni, [4]. The arc of the circle is limited by a chord of length $2x_H$. The height of the hill is equal to y_H. Knowing x_H and y_H, we can calculate the radius of the hill, R_H. We assume that the transmitted pulse approaches the receiver along one dominant ray, which is a creeping ray. The ray becomes "attached" the hill at attachment point Q'. Then it travels along the arc of the hill. The ray leaves the arc of the hill tangentially at the shedding point Q. Therefore the transmitted pulse experiences diffraction on the convex obstacle. The distortion of the pulse is determined by the transfer function over the distance Q'Q. The distortion caused by propagation in the air can be neglected.

The exact UTD expression for the amplitude term of frequency domain diffraction coefficient of one convex conducting obstacle for soft polarization is given by (1) [2, 3, 7], and the parameters of the model are indicated in Fig. 1 [4].

$$H_A(\omega) = -m\sqrt{\frac{2}{\beta}}\left(\frac{e^{-j\pi/4}\cdot\left(1-F(X_d)\right)}{2\xi_d\sqrt{\pi}} + P_s(\xi_d)\right), \qquad (1)$$

The diffraction coefficient (transfer function $H_A(\omega)$) given by (1) does not include the delay terms or spreading factor. The parameters and functions used in (1) are given in [2, 3]. The model of two cascaded convex obstacles is shown in Fig. 2.

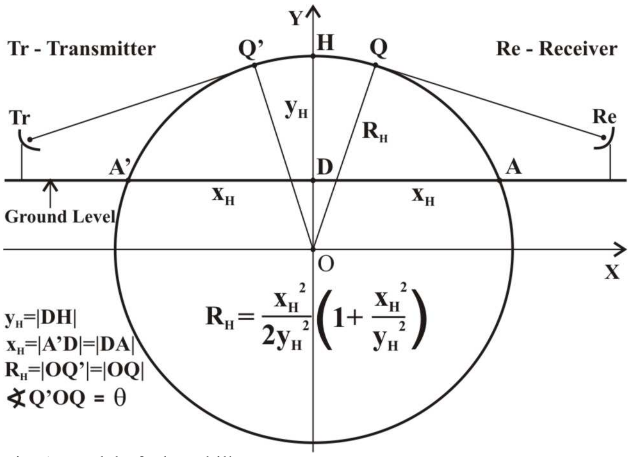

Fig. 1. Model of a bare hill.

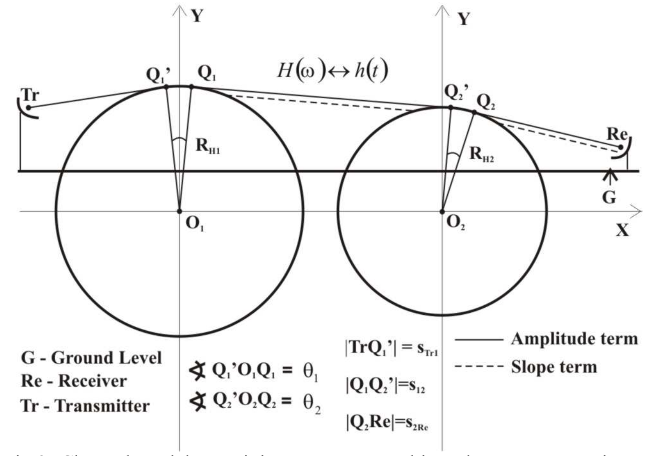

Fig.2. Channel model containing two convex objects between transmitter and receiver.

When two convex objects are taken into account, where the second object is in the transition zone of the first convex object, the frequency response of the "partial" channel has the following form:

$$[H_s(\omega,s)+H_A(\omega)]A(s)e^{-jk(\theta_1 R_{H1}+s)}, \tag{2}$$

where s is equal to $|Q_1Q_2'|$ in Fig. 2, A(s) is the spreading factor while the input of the channel is at point Q_1' in Fig. 2 and the output of the channel is at the point Q_2'.

The slope term and amplitude term of the frequency domain convex object diffraction coefficient are related through (3) [1].

$$H_s(\omega,s)=\frac{\partial H_A(\theta)}{s\partial\theta} \tag{3}$$

The Pekeris caret function - $P_s(\xi_d)$ - occurring in the second component of (1), which is given by (4):

$$H_{A2}(\omega)=-m\sqrt{\frac{2}{\beta}}P_s(\xi_d), \tag{4}$$

cannot be easily differentiated with respect to its argument. In order to do it we approximated $P_s(\xi_d)$ by the series of two forms. First of them is appropriate for smaller values of the argument ξ_d and the second one is applicable for bigger values of ξ_d. The threshold value of ξ_d is chosen in the way that the relative deviation of approximations of (4) cannot significantly exceed 0.1%.

The first of the approximating series (for bigger values of ξ_d) is the series expansion given by Keller [2, 3], Then

$$H_{A2}(\omega)\approx H_{A2>}(\omega)=\sum_{n=0}^{N}A_n\cdot\omega^{-1/6}\cdot e^{-\gamma_n\omega^{1/3}\theta}, \tag{5}$$

where:

$$A_n=\left(\frac{R}{2c}\right)^{1/3}\cdot j^{-1/6}\cdot\sqrt{\frac{c}{2\pi}}\cdot\frac{1}{[Ai'(q_n)]^2},\quad \gamma_n=-q_n\left(j\frac{R}{2c}\right)^{1/3}.$$

The second approximating series is related to smaller values of ξ_d (down to 0). In this situation we proposed to use the approximation of the Fock scattering function for the case of soft polarization for small ξ_d [5]:

$$p^*(\xi_d)\approx e^{j\pi/6}\sum_{n=1}^{N}\frac{\rho_n}{n!}\cdot j^{n/3}\cdot\xi_d^{\ n}, \tag{6}$$

The values of coefficients ρ_n can be found in [5, 6]. Then

$$H_{A2}(\omega)\approx H_{A2<}(\omega)=m\sqrt{\frac{2c}{\omega}}e^{-j\pi/4}\cdot \left(\frac{1}{2\xi\sqrt{\pi}}-e^{j\pi/6}\sum_{n=0}^{N}\frac{\rho_n}{n!}j^{n/3}\xi_d^{\ n}\right). \tag{7}$$

Using (5) and (7), the approximation of $H_{A2}(\omega)$ can be written in the following form:

$$H_{A2}(\omega)\approx\begin{cases}H_{A2>}(\omega) & \xi_d>\xi_{dT}\\ H_{A2<}(\omega) & \xi_d\le\xi_{dT}\end{cases}. \tag{8}$$

Now the question is what is the best threshold value of ξ_d, for which (4) can be approximated with sufficient accuracy (by this we mean that the relative deviation cannot exceed significantly 0.1% and at the same time the number of series terms is as small as possible). The calculations which we conducted have shown that this threshold value is about ξ_{dT} = 1.2. The closer the value of ξ_d is to the threshold, the bigger number of series terms is needed for appropriate approximation of (4). The maximum required number of terms in (5) and (7) equals 5 and 9, respectively. The more the value of ξ_d differs (in either way) from the threshold value ξ_{dT} = 1.2, the fewer term in (5) and (7) are needed, reaching at the limit only one term. The relative deviation of the approximation of (4) for 5 and 9 terms of the series is shown in Fig. 3.

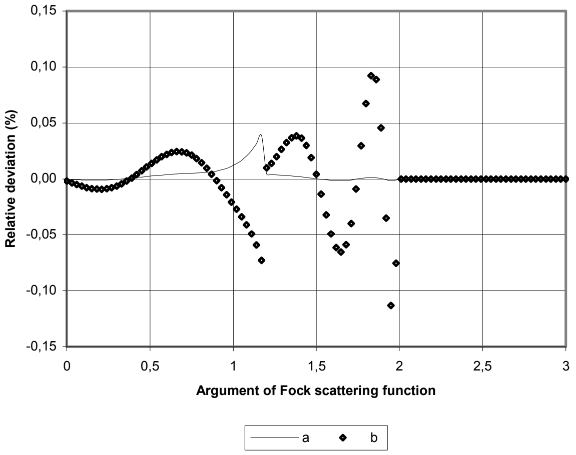

Fig. 3. Relative deviation of the approximation of (4) : a – Real part, b – Imaginary part

Having the approximation of the second part of the amplitude term of the convex obstacle diffraction coefficient we can find the second part of the slope term of this coefficient.

III. TD VESRION OF DIFFRACTION COEFFICIENT

In this section we show the TD version of (1), amplitude term of (2). It was derived in [8]. The TD version of the whole (2) was also derived in [8], but it is not necessary to show it here. For the sake of the presentation of the transformations given in the next section, showing only the form of TD amplitude term of (2), is sufficient to explain the benefits of this transformations. The TD slope term of (2) has similar form as the amplitude term. The TD version of (1) is given by

$$h_A(t) \approx \begin{cases} h_{A12}(t) + \mathrm{Re}\left\{ \overset{+}{h}_{A11}(t) + \overset{+}{h}_{A2<}(t) \right\} & \underset{f\in B}{\wedge} \xi_d(f) \le 1.2 \\ h_{A12}(t) + \mathrm{Re}\left\{ \overset{+}{h}_{A11}(t) + \overset{+}{h}_{A2>}(t) \right\} & \underset{f\in B}{\wedge} \xi_d(f) > 1.2 \end{cases}, \tag{9}$$

where:

$$\overset{+}{h}_{A11}(t) = -\frac{1}{\pi}\sqrt{\frac{c}{2\pi}}\frac{e^{-j\pi/4}}{\theta}\left(\frac{-1}{jt}\right)^{1/2}\Gamma\left(\frac{1}{2}\right), \quad h_{A12}(t) = \frac{L\theta}{2\pi\sqrt{2ct}\left(t+\frac{X}{c}\right)},$$

$$\overset{+}{h}_{A2>}(t) = \frac{1}{\pi}\sum_{n=0}^{N} A_n \frac{3}{(\gamma_n\theta)^{\frac{5}{2}}}\sum_{m=0}^{M}\frac{j^m}{m!}\Gamma\left(3m+\frac{5}{2}\right)\cdot\left[\frac{t}{(\gamma_n\theta)^3}\right],$$

$$B_n = (2c)^{\frac{1}{6}} R^{\frac{1}{3}} \frac{\rho_n}{n!}\left(\frac{R}{2c}\right)^{\frac{n}{3}} j^{\frac{n}{3}-\frac{1}{6}} \qquad \overset{+}{h}_{A2<}(t) \approx \frac{1}{\pi}\sum_{n=0}^{N} B_n\theta^n\left(\frac{-1}{jt}\right)^{\frac{n}{3}+\frac{5}{6}}\Gamma\left(\frac{n}{3}+\frac{5}{6}\right) + \frac{1}{\pi}\sqrt{\frac{c}{2\pi}}\frac{e^{-j\pi/4}}{\theta}\left(\frac{-1}{jt}\right)^{1/2}\Gamma\left(\frac{1}{2}\right),$$

and c is the velocity of electromagnetic wave propagation, B is the frequency band of the incident signal. The case of $\xi_d \le 1.2$ is adequate to the case of UWB signal transmitted in the baseband or to the geometry situations close to the grazing incidence with the higher frequency band occupied by an UWB signal. The case of $\xi_d > 1.2$ refers to the UWB signal transmitted in the higher frequency band in the geometry situations not close to the grazing incidence.

IV. IMPULSE RESPONSE OF TWO CONVEX OBJECTS

The considered channel model is shown in Fig. 2. We assume that the slope of the field is zero when the field originates from the transmitter. We further assume that the receiver is in the far zone, so the slope of the field originating from the second obstacle equals zero. Then the field at the receiver is defined in the time domain by (10) [8] (the time delay of the signal over the distance $\theta_{1/2}R_{1/2}$, Fig 2, is not included):

$$E_{\mathrm{Re}}(t) = \Big(E_2(t) * h_A(t, L_{A12\,\mathrm{Re}}, R_2, \theta_2) + \frac{\partial E_2(t)}{\partial n} * d(t, L_{s12\,\mathrm{Re}}, R_2, \theta_2)\Big) A_2(s) * \delta\left(t - \frac{s_{2\,\mathrm{Re}}}{c}\right), \tag{10}$$

$$E_2(t) = E_1(t) * h_A(t, L_{ATr12}, R_1, \theta_1) * \delta\left(t - \frac{s_{12}}{c}\right) A_1(s), \tag{11}$$

where $E_1(t)$ is the transmitted signal, $d((t, L_{s12Re}, R_2, \theta_2)$ is the TD version of (12)

$$D(L_{S12\,\mathrm{Re}}, R_2, \theta_2) = \frac{s_{2\,\mathrm{Re}} H_s(L_{sTe12}, R_1, \theta_1, s_{2\,\mathrm{Re}})}{jk}, \tag{12}$$

Equation 10 is the sum of operations of convolutions of the slope of the field and the amplitude term of the field after the first obstacle with the slope term and amplitude term of the time domain second convex obstacle diffraction coefficient respectively. The slope term and the amplitude term of the field after the first obstacle is found with the usage of he first convex obstacle diffraction coefficient.

Now we will present how we solved the convolution operations occurring in (10), precisely the convolution operation of the components occurring in time domain convex obstacle diffraction coefficient (impulse response presented section 3): $h_A(t)*h_S(t,s)$, $h_A(t)*h_A(t)$ and $h_S(,st)*h_S(t,s)$. Some of the expressions occurring in $h_A(t)$ and $h_S(t,s)$ have singularities at argument t=0. For this reason the convolutions cannot be solved directly in an analytical way. For the purpose of solving this convolutions we propose to use the following relationship:

$$p(t) * \int_0^t h(\tau)d\tau = \int_0^t p(\tau) * h(\tau)d\tau \tag{13}$$

Using (13) we can find the convolution of the terms occurring in $h_A(t)$ and $h_S(t,s)$ if only the integrants of finite orders of this terms do not have singularities. When such integrants are found, the convolutions of the terms of $h_A(t)$ and $h_S(t,s)$ can be derived by an operation of a derivation of the order equal to the sum of the orders of the operations of integration which were applied to the convoluted components of $h_A(t)$ and $h_S(t,s)$.

The $h_A(t)$ and $h_S(t,s)$ include components of the following form:

$$C_1(t)=\frac{1}{\sqrt{t(t+a)}},\quad C_2(t)=\frac{\partial}{\partial t}\left(\frac{1}{\sqrt{t(t+a)}}\right),\quad C_3(t)=\frac{1}{\sqrt{t}},\quad C_4(t)=\frac{1}{\left(\sqrt{t}\right)^3},\quad C_{5n}(t)\frac{1}{t^{\frac{n}{3}+\frac{5}{6}}}. \tag{14}$$

The first order integrant of $C_1(t)$ and the second order integrant of $C_2(t)$ have the form:

$$C_{1/2i}(t) = a\tan\left(\sqrt{bt}\right), \tag{15}$$

where

$$b=\frac{2c}{L\theta^2},$$

the first order integrant of $C_3(t)$ and the second order integrant of $C_4(t)$ have the form:

$$C_{3/4i}=\sqrt{t} \tag{16}$$

and the kth order integrant of $C_{5n}(t)$ has the form:

$$C_{5ni}=t^{-\frac{1}{6}-\frac{2n}{3}+k} \tag{17}$$

where

$$k=\left[\frac{n}{3}+\frac{5}{6}\right]+1$$

while [x] means the biggest integer value not bigger than x.

The convolutions of (16) and (17) can be easily obtained, but the operations of convolutions with (15) requires approximation of (15) to the form convenient for integration. We propose to approximate (15) by the functions of two forms dependent on the value of the argument of (15). For arguments not bigger than x_T (15) can be approximated by the function:

$$C_{1/2i\_ap<}(t)=a_1 t+a_0\sqrt{t} \tag{18}$$

For arguments bigger than x_T we propose to approximate (15) by the formulation:

$$C_{1/2i\_ap>}(t)=a_3+\frac{a_2}{\sqrt{t}} \tag{19}$$

We find coefficients a_n (n=0..3) through the usage of optimisation algorithm (for example genetic algorithm). We distinguish two sets of optimal a_n coefficients. First of them relates to the situation when only one expression of the form (15) occurs in the operation of convolution. The second set of coefficients is needed when in the operation of convolution are two expressions of the form (15). The difference in the goals of the optimisation for the case of one and two expression of the form (15) occurring in the operation of convolution concerns the definition of the approximation error. For the case of one expression of the form (15) we minimize the approximation deviation of one such term, while for the case of two expression of the form (15) we minimize the overall deviation of convolution of two such terms, because the relative errors of approximation of individual terms add in the process of multiplying. In result we have in the first case and in the second case 4 and 8 optimal values of a_n respectively, as well as the optimal threshold value of x_T. Through the application of the genetic algorithm, written by us, we found the following values of a_n coefficients and x_T, for which the relative errors of solving the operations of convolution occurring in (10) do not exceed 2%:

$$a_0=1.014\quad a_1=-0.234\quad a_2=-\frac{\pi*0.621}{2}\quad a_3=\frac{\pi}{2},$$

x_T=4.2 for the case of one expression of the form (15) and

$$a_{0(1)}=1.161\quad a_{1(1)}=-0.176\quad a_{2(1)}=-0.936\quad a_{3(1)}=1.581$$

$$a_{0(2)}=0.907\quad a_{1(2)}=-0.269\quad a_{2(2)}=-0.948\quad a_{3(2)}=1.562$$

x_T=3.6 for the case of two expressions of the form (14).

Having the values of a_n and x_T we can solve analytically the operations of convolutions occurring in (10).

V. VERIFICATION OF THE DERIVED IMPULSE RSPONSE

In this section we examine the accuracy of the results obtained through the usage of the method presented in previous sections. In order to verify the accuracy of formula (10) with the simplification described in section 4 we examine the distortion of an UWB pulse caused by a two cascaded convex obstacles. It is done through solving (10) with substituting as $E_1(t)$ the particular UWB pulse p(t). The pulse p(t) has two parameters, a and t_c, which are the width of the pulse and the middle argument of the pulse, respectively.

$$p(t)=\left[1-4\pi\left(\frac{t-t_c}{a}\right)^2\right]e^{-2\pi\left(\frac{t-t_c}{a}\right)^2} \tag{20}$$

The obtained results are compared with the results of frequency domain calculations obtained by using IFFT (Fig.

4) The parameters of the convex objects are: θ_1=0.15, θ_2=0.20, R_1=200m, R_2=150m and are noticed in Fig.2.

Fig. 4 shows the distorted pulse and the normalized transmitted pulse with its parameters set to the following values: a = 3ns, t_c = 1ns. We see that the accuracy of the results obtained with the method used by us is very good.

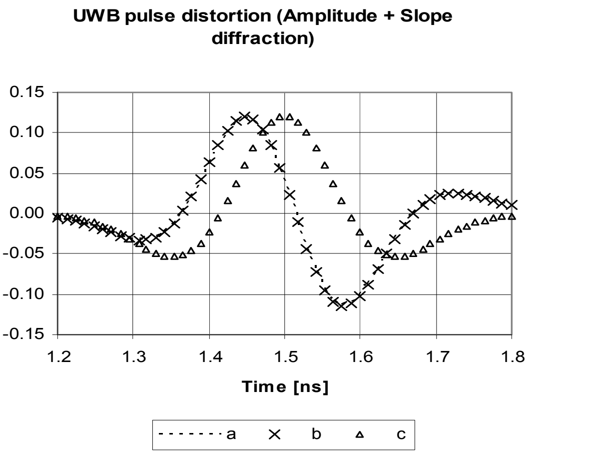

Fig. 4. The shape of the distorted UWB pulse with the parameters: a=3ns, t_c=1.ns. Comparison of the results of the pulse distortion obtained trough IFFT (a) with the results of the pulse distortion obtained through direct time domain calculations (b), incident UWB pulse normalized to the amplitude of the distorted pulse (c). The values of parameters of the convex objects are: θ_1=0.15, θ_2=0.20, R_1=200m, R_2=150m.

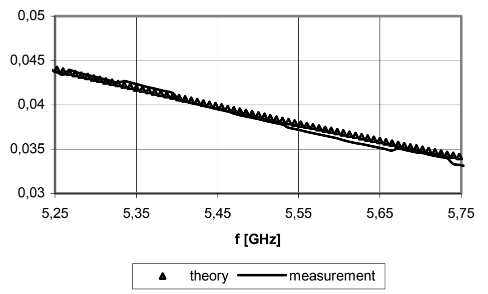

Fig. 5. Theoretical and measured squared amplitude of one rounded pillar with 0.4 m diameter.

In Fig. 5 we compare the frequency domain theoretical results with the results of measurements of the UWB impulse propagation on non perfectly conducting convex pillar with the diameter 0.4 m. The impulse was created according to the WiMedia standard. In the figure there is shown the theoretical and measured square amplitude (the square of the amplitude part of the frequency response) of one such pillar. We assumed that the input signal went to the receiver along two creeping rays.

VI. CONCLUSIONS

In the paper we present the way for obtaining the time domain formulation for an amplitude and a slope term of a diffraction coefficient of a convex obstacle. The theoretical TD formulations are in very good agreement with the results of IFFT and the results of measurements. Although the derived formulations are applicable for a limited range of channel parameters [9], it is still valid for a numerous channel characteristics. There is still work to be done concerning extending of the time domain diffraction coefficient formulations for the whole range of channel parameters, co composed of not good conducting materials. Although (10) relates to soft polarization case alternative formulas can be derived for hard polarization case using similar method to that applied for soft polarization case.

REFERENCES

[1] G. Koutitas, "A UTD Solution for Multiple Rounded Surfaces", IEEE Transactions on Antennas and Propagation, vol. 54, no. 4, April 2006, pp 1277-1283.
[2] D. A. McNamara, "Introduction to the uniform geo- metrical theory of diffraction", Artech House, Boston, London, 1990.
[3] P. H. Pathak, An asymptotic analysis of the scattering of plane waves by a smooth convex surface, Radio Science, vol. 14, no. 3, May-June, 1979, pp 419-435.
[4] H. L. Bertoni, "Radio propagation for modern wireless systems", Upper Saddle River, N. Jersey, 2000.
[5] N. A. Logan, "General research in diffraction theory", Lockheed Misiles and Space Division, Tech. Rep. LMSD-288087, 1959.
[6] P. H. Pathak, "A Time Domain Formulation of the Uniform Geometrical Theory of Diffraction for Scattering from a Smooth Convex Surface", IEEE Transactions on Antennas and Propagation, vol. 55, no. 6, June 2007, pp 1522-1534.
[7] P.Górniak, W. Bandurski, "UWB pulse distortion in the channel containing cylindrical hill", European Conference on Antennas and Propagation, Nice 6-10 November, 2006.
[8] P.Górniak, W. Bandurski, "Direct Time Domain Analysis Of An UWB Pulse Distortion By Convex Objects With The Slope Diffraction Included" IEEE Transactions on Antennas and Propagation, Vol. 56, September 2008, pp 3036-3044.