System Fidelity Factor in analysis of inverter-interconnect-inverter VLSI systems

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Abstract

The paper presents the method to classify of the shape of the clock signal distortions and rough estimation of a clock skew by means of system fidelity factor (SFF). The method is based on cross-correlation coefficient calculations for periodic clock signal. The signals are represented by their Fourier series. The steady state in clock distribution networks is assumed. In the paper is presented the formula for SFF coefficient and an example of its calculation.

Introduction

The Clock Distribution Network (CDN) is one of the most performance limiting structures in a high-speed chip. The clock jitter and skew are the main limiters of the cycle time and the data rate of high speed I/Os.

In deep submicrometer integrated circuits, e.g. CND, interconnect delay dominates gate delay. Furthermore, wire inductance can no longer be ignored, due to higher signal frequencies and longer wire lengths. Accurate and efficient resistance–capacitance–inductance (*RLC*) interconnect models are, therefore, critical in the design of high-performance integrated circuits. In this paper analysis of CDN interconnect is based on a Fourier series approach to a periodic input signal like in [2, 3]. No approximation is made to the transfer function of the interconnect. The far-end response is approximated by the summation of several sinusoids.

There exist frequently necessity to investigate the influence of system parameter on the shape of the output signal with reference to input signal. Such a situation occurs in inverter-interconnect-inverter CND systems. We want to know for which values of parameters the output signal has the shape closest to the input signal. Usually in that case we simulate system for a few values of parameters and next we compare the output signal with input signal or/and output signals between each other. Such a comparison rely on qualitative choice of the "best" shape and looking the parameters for which it was happened. There exist, of course, quantitative methods of comparing signal shapes such delay or clock skew calculation, but in the case of large number of parameters which can take large range of levels, the set of data is huge and comparison is very difficult and time consuming. Hence there exist a need to have one simple and easy to calculate measure of the distortion of the output signal and to classify them. As such a tool can be considered cross-correlation coefficient between output and input signals in time domain.

Such an approach has been used recently in [1] to compare UWB output and input signals of the two antennas system . In [2, 5] skew and overshoot time was calculated with that aim, but was not used to compare input and output signals for a large number of parameter values. The main goal of this paper is to propose System Fidelity Factor (SFF) by means of cross-correlation coefficient. Knowledge about such factor (SFF) is very useful, because one can analyzes the influence of model parameters such as RLC-per unit of length parameters and gate parameters such as R_s and C_0 on shape of the clock signal in inverter-interconnect-inverter system. More over we can choose the optimal electrical parameters, which can be transformed next into geometrical dimension.

The paper is organized as follows. First we introduce the VLSI system inverter - interconnect – inverter and the main idea of output response calculation in steady state. In the third and fourth sections the SFF factor is presented. In the fifth section we present the method of using it. In the next section the simulation result are presented. We conclude in the last section.

Clock Signal Analysis

The equivalent circuit for the transmission line model of the interconnect is shown in Fig.1.

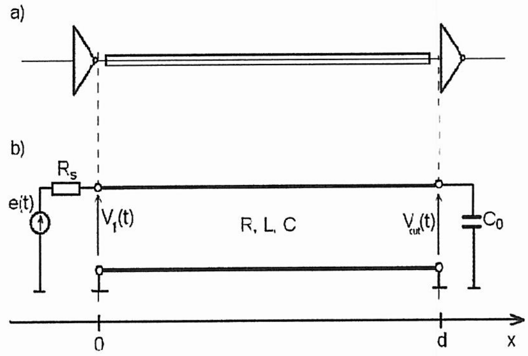

Fig. 1 The system inverter-interconnect-inverter.

The transfer function from the input to the far end of the interconnect system can be written as:

$$H(j\Omega)=\frac{\sqrt{1+\frac{\varepsilon}{j\cdot\Omega}}}{\beta+\sqrt{1+\frac{\varepsilon}{j\cdot\Omega}}}\frac{e^{-\gamma y}+\rho_0 e^{-\gamma(2l-y)}}{1-\rho_0\rho_w e^{-2\gamma l}}, \tag{1}$$

where:

normalized frequency $\Omega=\omega\cdot T_d,\quad T_d=d\sqrt{LC}$,

$$\varepsilon=\frac{R_t}{Z_o},\quad \beta=\frac{R_s}{Z_o},\quad \gamma=j\cdot\Omega\cdot\sqrt{1+\frac{\varepsilon}{j\cdot\Omega}},$$

$$\rho_w=\frac{\beta-\sqrt{1+\frac{\varepsilon}{j\cdot\Omega}}}{\beta-\sqrt{1+\frac{\varepsilon}{j\cdot\Omega}}},\quad \rho_0=\frac{\frac{\alpha}{j\cdot\Omega}-\sqrt{1+\frac{\varepsilon}{j\cdot\Omega}}}{\frac{\alpha}{j\cdot\Omega}-\sqrt{1+\frac{\varepsilon}{j\cdot\Omega}}},$$

$$\alpha = \frac{C_t}{C_0},\quad Z_o = \sqrt{L/C},\quad C_t = C \cdot d, etc..$$

The clock signal has trapezoidal waveform as it is shown in Fig.2. Hence the steady state response, voltage on the C_0 capacitance, has form:

$$V_{out}(t) = \sum_{m=-\infty}^{\infty} F_m H(j\Omega_0 m) e^{j\Omega_0 mt}, \tag{2}$$

where:

$$\Omega_0 = 2\pi / T_0,$$

$$F_m = \frac{-A\left[1 - e^{-j\Omega_0 m T_r} - e^{-j\Omega_0 m (T_r + T_s)} + e^{-j\Omega_0 m (2T_r + T_s)}\right]}{(\Omega_0 m)^2 T_r T_0},$$

$$H(j\Omega) = A(\Omega) e^{j\varphi(\Omega)}.$$

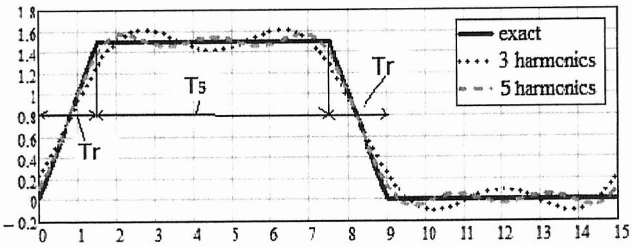

Fig. 2 The input signal, and first harmonics of the output signal.

Analyzing the infinite series expansion of the trapezoidal waveform (2) we can observe [2, 5] that the impact of the odd harmonics is significantly larger than the impact of even harmonics. More over it is well known that a complete symmetric clock signal with 50% duty cycle does not exhibit any even order harmonics. But also for duty cycle between 45-50% [2, 5] the quality of the approximation is also acceptable.

Cross-correlation coefficient

Calculation of the cross-correlation coefficient is well known procedure in signal processing area e.g. [6]. Definition of cross-correlation coefficient for real valued periodic signals is following [6]:

$$\rho_{x,y}(t) = \frac{1}{T}\int_0^T x(t+\tau) y(\tau) d\tau. \tag{3}$$

Assuming that signals x(t) and y(t) have Fourier series representation:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t},\quad y(t) = \sum_{k=-\infty}^{\infty} Y_k e^{jk\omega_0 t}, \tag{4}$$

we have

$$\rho_{x,y}(t) = \sum_{k=-\infty}^{\infty} X_k Y_k^* e^{jk\omega_0 t}. \tag{5}$$

Normalized cross-correlation coefficient can be define as

$$\rho^N_{x,y}(t) = \frac{\frac{1}{T}\int_0^T x(t+\tau) y(\tau) d\tau}{\sqrt{\frac{1}{T}\int_0^T x(\tau)^2 d\tau}\sqrt{\frac{1}{T}\int_0^T y(\tau)^2 d\tau}},$$

or

$$\rho^N_{x,y}(t) = \frac{\sum_{k=-\infty}^{\infty} X_k Y_k^* e^{jk\omega_0 t}}{\sqrt{\sum_{k=-\infty}^{\infty} |X_k|^2}\sqrt{\sum_{k=-\infty}^{\infty} |Y_k|^2}}. \tag{6}$$

System Fidelity Factor

Now we can introduce, as in [1], System Fidelity Factor (SFF) by

$$SFF = \max_n \left[\rho^N_{x,y}(t_n)\right],\quad t_0 t_1, \ldots t_n \in T_o. \tag{7}$$

We identify x(t) as an input signal and y(t) as an output signal of the inverter-interconnect-inverter system. The transfer function of system is $H(j\Omega_n) = Y_n/X_n$. Finally SFF has following form:

$$SFF = \max_n \left[\frac{\sum_{k=-\infty}^{\infty} |X_k|^2 H_k e^{jk\omega_0 t}}{\sqrt{\sum_{k=-\infty}^{\infty} |X_k|^2}\sqrt{\sum_{k=-\infty}^{\infty} |X_k|^2 |H_k|^2}}\right]. \tag{8}$$

SFF described by formula (8) fulfill the following inequality: $0 \leq SFF \leq 1$. Father we will use approximate version of (8) in the form:

$$SFF(K) = \max_n \left[\frac{\sum_{k=-K}^{K} |X_k|^2 H_k e^{jk\omega_0 t}}{\sqrt{\sum_{k=-K}^{K} |X_k|^2}\sqrt{\sum_{k=-K}^{K} |X_k|^2 |H_k|^2}}\right]. \tag{9}$$

Transfer function $H(j\Omega_n)$ depends on parameters α, β, ε which in turn depend on per unit length line parameters and R_s, C_o (see Fig.1). SFF(K) also depends on the above parameters. Next we will investigate how the value of SFF(K) characterize the shape of output signal with respect to original.

Scaling of system fidelity factor (SFF)

Knowledge of parameters α, β, ε and additionally e.g. R_s, C_o permits for finding the rest of parameters:

$$C_t = \alpha \cdot C_0,\quad Z_o = \beta R_s,\quad L_t = (Z_o)^2 \alpha C_0,\quad R_t = \varepsilon Z_o.$$

Scaling SFF should be done "experimentally" by simulations. Assume the following ranges of parameters:

$$0 < \alpha \leq 10,\quad 0 < \beta \leq 5_s,\quad 0 < \varepsilon \leq 1,$$

every range is divided to M = 20 sections. In this way we obtain the space of data 20x20x20. For every combination of parameters we calculate SFF(K) on time interval $3T_o$ (T_o=8).

The results – SFFs as a function of ε_i $1 \le i \le 20$ are shown in Fig.3.

The minimal value SFF (0.987) (Fig.1) is reached for i =10 (ε_{10}= 0.5) and α = 1.5, β=0.5 for times τ={1.6, 9.6, 17.6}. The maximal value (0.99) SFF is reached for i =2 (ε_2= 0.1) and α = 1.5, β=0.75 for times τ={1.6, 9.6, 17.6}. Output and input

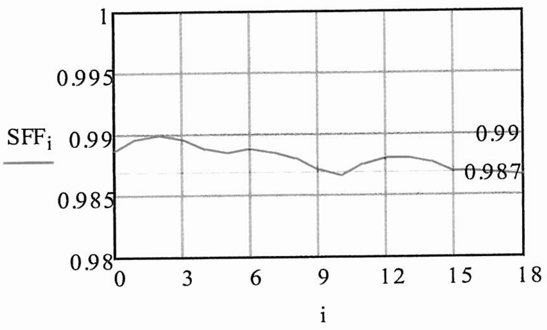

Fig.3 Values of SFF(15) for ε_i= i/20 and $0 < \alpha \le 10, \quad 0 < \beta \le 5$

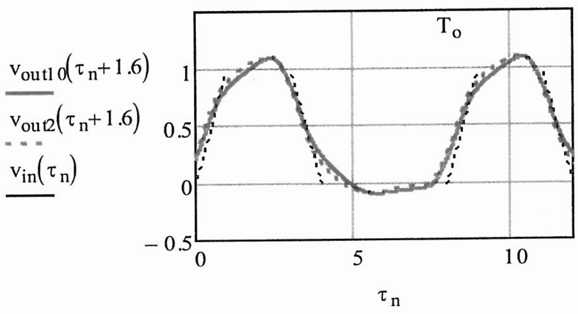

Fig.4 Output and input waveforms for minimal $v_{out10}(t)$ and maximal $v_{out2}(t)$ values of SFF(15)

waveforms for the minimal and maximal values of SFF(15) are shown in Fig. 4. One can see that the output signal is considerably distorted, however the slope at the 50% threshold level is almost the same as in input signal. Moreover the $v_{out2}(t)$ and $v_{out10}(t)$ are practically undistinguishable.

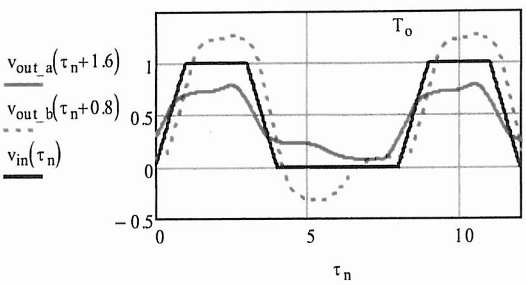

Fig.5 Output and input waveforms $v_{out\_a}(t)$ and maximal $v_{out\_b}(t)$ for SFF(15) = 0.95

For smaller values of SFF, for example for 0.95 it can be found that the same values of SFF can be reached for α_1= 4.5, β_1=0.5, ε=0.15 $v_{out\_b}(t)$ and for α_2 = 3.5, β_2=1.75, ε=0.15 $v_{out\_a}(t)$. The output voltages $v_{out\_a}(t)$ and $v_{out\_b}(t)$, shown in Fig.5, differ considerably. Basing on further simulations it was found the threshold value of SFF = 0.98 (e.g. for α_1= 3, β_1=0.55, ε=1 $v_{out\_c}(t)$ and for α_2 = 5.5, β_2=0.75, ε=1 $v_{out\_d}(t)$) above which even for different sets of values α, β, ε the corresponding waveforms are very similar as it is shown in Fig.6. Hence one can accept the value of SFF=0.98 as a threshold over which the output signal has satisfactory quality.

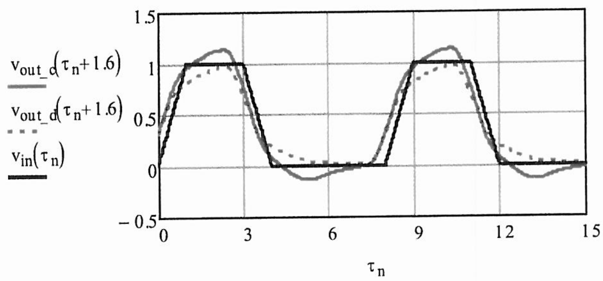

Fig.6 Output and input waveforms $v_{out\_c}(t)$ and maximal $v_{out\_d}(t)$ for SFF(15) = 0.98

Evaluation of SFF for interconnects

Now we can apply SFF to classify distortions caused by considered interconnect system (Fig.1) with respect to input trapezoidal waveform. The parameters of the model are taken from [1]. In the Table 1, are present the calculation results. As result we consider the values of SFF and the time (t_{max}) at which given SFF value was reached first time (during the period).

Table 1

Calculated coefficient SFF and t_{max} . Exact clock skew (SPICE) and exemplary circuit parameters taken from [1], *w*-width of interconnect, R_s, C_o see Fig.1.

w [μm]	R_s	C_o	SFF	t_{max} [ps]	exact [ps]
2	20	50	0.943	30	28.8
2	60	100	0.996	40	39.9
2	100	500	0.971	75	73.4
6	20	50	0.948	35	41.8
6	40	100	0.991	45	45.3
6	60	500	0.992	70	68.6
10	20	50	0.959	40	47.2
10	40	100	0.997	50	53.0
10	60	500	0.997	75	74.4

The SFF has been calculated by means of formula (9) using K=10 harmonics. The time t_{max}, at which normalized cross-correlation coefficient (6) takes maximal value first time, was found by looking up the vectors containing all values of $\rho^N_{xy}(t_n)$ and time moments $t_0\, t_1,\ldots t_n \in T_o$ as in formula (7).

We present some simulations results in Fig. 7 - 9 for each values of w, R_s, C_o from Table 1. In the Fig.7 we can see that the waveform $v_{out}(\tau,1)$ for SFF=0.996 has the shape closest to original $v_{in}(\tau)$ and t_{max} is very close to exact clock skew shown in Table 1. The same observation can be done for Fig.8. The curve for SFF=0.992 has the shape closest to original one and t_{max} has value close to exact clock skew from Table 1. Finally in Fig.8 the curve for SFF=0.997 has the shape closest to original one and t_{max} has value close to exact clock skew from Table 1. It can be noticed in every of the considered cases, that t_{max} is closest to exact clock skew time for the greatest SFF value. The maximal and minimal errors for clock skew estimations by t_{max} are 16.268% and 0.251% respectively. The average error in Table 1 is 5.254%.

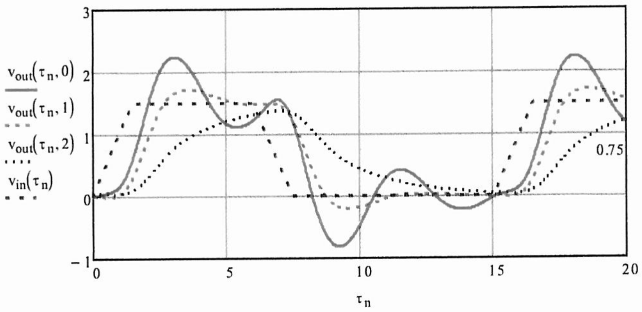

Fig.7 Output waveforms: $v_{out}(t,0)$ (SFF=0.943, α= 7.2, β= 0.216, ε=0.104), $v_{out}(t,1)$ (SFF=0.996, α=3.6, β= 0.649, ε= 0.104), $v_{out}(t,2)$ (SFF=0.971, α=0.72, β= 1.082, ε= 0.104), and input $v_{out}(t,0)$, T_d=33.28ps

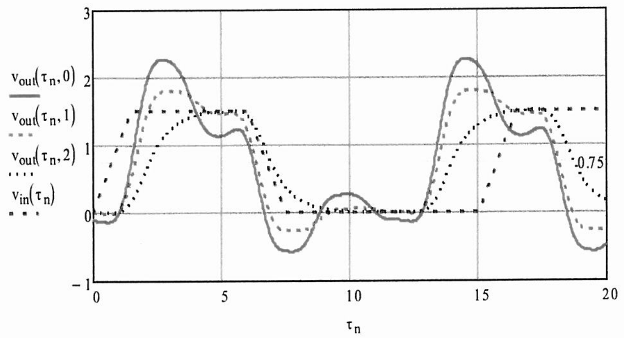

Fig.8 Output waveforms: $v_{out}(t,0)$ (SFF=0.948, α= 13.2, β= 0.312, ε=0.191), $v_{out}(t,1)$ (SFF=0.991, α=6.6, β= 0.623, ε= 0.191), $v_{out}(t,2)$ (SFF=0.992, α=1.32, β= 0.935, ε= 0.191), and input $v_{out}(t,0)$, T_d=42.37ps

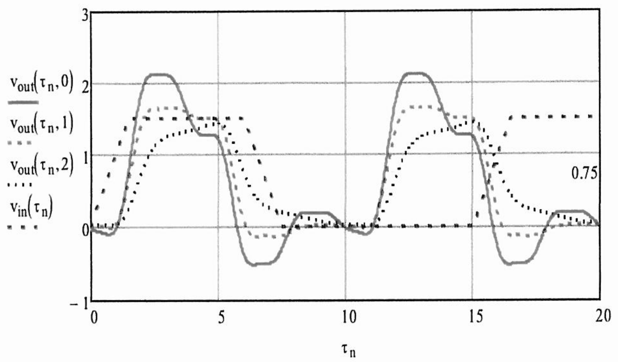

Fig.9 Output wave forms: $v_{out}(t,0)$ (SFF= 0.959, α= 19.6, β= 0.394, ε=0.191), $v_{out}(t,1)$ (SFF= 0.997, α= 9.8, β= 0.789, ε= 0.191), $v_{out}(t,2)$ (SFF= 0.988, α= 1.96, β= 1.183, ε= 0.191), and input $v_{out}(t,0)$, T_d=49.7ps

Conclusions

In the paper is presented a tool to detect the maximal and minimal distortion of clock signal in CDN. This tool permits for a fast valuation of interconnect parameters influence on the shape of the clock signal. Moreover this tool gives a rough estimation of the clock skew. The approach is based on the cross-correlation coefficient calculation. Calculations are very fast. Presented approach has been used to estimate a simple case of interconnect system. However it can be used to complicated clock tree under condition that its transfer function is known and steady state can be is assumed.

References

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